TRINITY COLLEGE

Mathematics Methods 3,4
Test 6 2017

Calculator Assumed

Sampling & Confidence Intervals

STUDENT'S NAME MARKING KEY

DATE: Thursday 31st August **TIME:** 50 minutes **MARKS**: 51

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

Marine biologists want to determine if a local species of fish is growing to a smaller size than it used to. They collect and measure 90 of the 250 fish known to exist in the large pond.

(a) Is the sample size appropriate? Comment why/why not. [2]

Yes, $n > 30$ so we can approximate using the normal distribution.

(b) State two additional pieces of information would make us more confident that the sample is a close representative of the population? [2]

- Two appropriate responses regarding avoiding bias

2. (3 marks)

A census for a particular country showed that 34.8% of people used public transport at some point during a regular week. At about the same time as the census, a sample of people in a region of the country showed that 1573 of those people used public transport at some point during a regular week.

(a) Given $\hat{p} = 0.286$, determine n, the number of people sampled. [1]

$n = 5500$

(b) A 90% confidence interval for $\hat{p}$ was calculated and determined to be $0.276 \leq \hat{p} \leq 0.296$. Comment on the statistical significance of the sample point estimate. [2]

The sample is not statistically significant as p does not lie within the confidence interval.

3. (4 marks)

28% of orange trees never bear fruit. A wholesaler purchases 126 trees. Use sample proportion techniques to determine the probability that at least one third of these trees will not bear fruit. Show any distributions used.

$p = 0.28$

$\sigma = \sqrt{\dfrac{0.28(1-0.28)}{126}}$

$= \dfrac{1}{25}$

$\hat{p} \approx N\left(0.28, \dfrac{1}{25}^2\right)$

$P(\hat{p} \geq \frac{1}{3}) = 0.0912$

or $X \sim B(126, 0.28)$

$P(X \geq 42) = 0.1097$

4. (9 marks)

An NBN survey randomly selected Australian homes to ascertain the number of homes that still have a landline phone. It was found that of the 564 homes surveyed, 223 still had a landline phone.

(a) Briefly explain a possible random sampling method used to data for the survey. [1]

Any appropriate response

(b) Determine the standard deviation of the random variable $\hat{p}$, for samples of 564 homes. [2]

$$\sigma = \sqrt{\frac{0.3954(1-0.3954)}{564}}$$

$$= 0.0206$$

(c) State a 90% confidence interval for the sample proportion. [3]

$z = 1.645$

standard error $= 0.0338$

90% C.I $= (0.3615, 0.4293)$

(d) A further sample of homes resulted in a sample proportion of 0.38. If the 95% confidence interval resulted in a margin of error less than 0.05, determine the minimum sample size of this sample. [3]

$$e = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$z = 1.96$

$$0.05 = 1.96\sqrt{\frac{0.38(0.62)}{n}}$$

$n = 362.02$

$n \sim 362$ homes.

363

5. (9 marks)

There are over 600 000 commercial trucks in Australia. According to recent research, 86% of commercial trucks in Australia satisfy the criteria to be called roadworthy.

(a) Explain why it is likely that the research involved some form of random sampling. [1]

There are too many trucks to survey $\rightarrow$ not financially / logistically possible.

(b) If a large number of random samples of 35 commercial trucks are collected, what proportion of these samples are expected to contain less than 80% of roadworthy trucks? [3]

$n = 35$

$p = 0.86$

$\sigma = \sqrt{\frac{0.86(0.14)}{35}}$

$= 0.05865$

$\hat{p} \sim N(0.86, 0.05865^2)$

$P(\hat{p} < 0.8) = \underline{\underline{0.1531}}$

(c) If a large number of random samples of 200 commercial trucks are collected, what proportion of these samples are expected to contain between 85% and 90% of roadworthy trucks? [3]

$n = 200$

$p = 0.86$

$\hat{p}_2 \sim N(0.86, 0.02454^2)$

$\sigma = \sqrt{\frac{0.86(0.14)}{200}}$

$= 0.02454$

$P(0.85 < \hat{p}_2 < 0.90)$.

$= \underline{\underline{0.6067}}$

(d) One of the answers to (b) or (c) should be treated with some caution. State which answer and explain why. [2]

(b) as the sample size is far smaller to that of (c).

6. (5 marks)

A student simulated taking a random sample from a population in which it was known that one out of three people had never been overseas. He then calculated a 90% confidence interval for the population proportion of people who had never been overseas. The confidence interval found was (0.20, 0.46)

He then repeated this process for 9 more random samples. The 90% confidence interval for each proportion was then calculated. The intervals obtained by the student after repeating the simulation nine more times are shown below, rounded to two decimal places.

(0.18, 0.43) (0.26, 0.52) (0.15, 0.40) (0.11, 0.34) (0.09, 0.30) (0.28, 0.55)

(0.23, 0.49) (0.26, 0.52) (0.31, 0.58)

(a) Determine, with reasoning, whether this group of confidence intervals reflect the population from which the samples are drawn. [2]

Yes, as the pop. proportion $p = \frac{1}{3}$ is within the bounds of 9 of the 10 confidence intervals.

(b) The margin of error for the first interval, (0.20, 0.46) is 0.13.

Calculate the margin of error for the interval (0.11, 0.34), then explain why this is different to the margin of error for the first interval. [3]

$$e = \frac{0.34 - 0.11}{2}$$
$$= 0.115$$

A different sample is likely to produce a new $\hat{p}$ which in turn creates a different value for σ and different C.I.

7. (10 marks)

A population of 100 people who worked at a local shopping centre and their views on whether Perth should lengthen trading hours on weekends is shown below.
The letter Y indicating the person would vote "Yes" in a referendum and N indicating the person would vote "No".

The population has been arranged below for your convenience.

1 Y	Y	Y	~~Y~~	Y	Y	Y	~~Y~~	Y	~~Y~~
11 Y	~~Y~~	Y	Y	Y	~~Y~~	~~Y~~	~~Y~~	Y	Y
21 Y	Y	Y	~~Y~~	~~Y~~	Y	Y	Y	Y	Y
31 Y	Y	Y	~~Y~~	Y	Y	N	~~N~~	~~N~~	N
~~41 N~~	~~N~~	~~N~~	N	N	N	N	N	N	N
~~51 N~~	~~N~~	N	~~N~~	~~N~~	N	N	N	N	~~N~~
~~61 N~~	N	N	N	~~N~~	N	N	N	N	~~N~~
71 N	N	N	~~N~~	N	N	N	N	N	N
~~81 N~~	N	N	~~N~~	N	N	N	N	N	N
91 N	~~N~~	N	N	N	N	~~N~~	N	~~N~~	~~100 N~~

A Year 12 methods student took a random sample of 30 workers using his Classpad to generate the members of the sample group.

(a) The output of the student's Classpad is shown below. On the grid above, indicate with an X the members of the sample group. [2]

randList(30, 1, 100)

{43, 70, 61, 34, 18, 16, 92, 74, 81, 24, 84, 39, 42, 99, 25,

55, 12, 100, 51, 65, 97, 17, 54, 52, 38, 10, 8, 41, 4, 60}

The student wishes to estimate how many workers would vote "Yes" at a referendum on trading hours using the sample.

(b) Determine $\hat{p}$, the point estimate for the number of "Yes" voters in the sample. [1]

$$\hat{p} = \frac{10}{30}$$

$$= \frac{1}{3}$$

(c) Calculate a 95% confidence interval for the point estimate in (b). [3]

$Z = 1.96$

$se = 0.1687$

$\therefore$ 95% C.I $= (0.1646, 0.502)$

(d) Given that the population proportion (p) is known, comment on the validity of the sample taken. [2]

$p = \frac{36}{100}$

$= 0.36$ is well within the 95% C.I

So the sample is statistically valid.

(e) A second sample of the same population taken by a different methods student indicated the 99% confidence interval to be (0.12, 0.56).
How many workers in the sample do we expect to vote yes? [2]

between 4 and 17 people

or 11 people.

4 to 17. ✓

or 11. ✓✓

8. (7 marks)

In 2014, a random sample of 125 g packets of Pops Pork Crackle, was taken to determine the proportion of packets that were under 125 g (i.e. underweight). The resultant 97% confidence interval for the proportion of underweight packets was (0.165953, 0.234047).

(a) Determine the number of 125 g packets of Pops Pork Crackle in the sample that were under 125 g. [4]

$$\hat{p} = \frac{0.165953 + 0.234047}{2}$$

$$= 0.2$$

For 97% C.I $z = 2.17$

$$2.17\sqrt{\frac{0.2(1-0.2)}{n}} = 0.2 - 0.165953$$

$$n = 650$$

$\therefore$ number of packets < 125 g

$= 650 \times 0.2$ $= \underline{\underline{130}}$ packets

(b) A more recent sample of 125 g packets of Pops Pork Crackle found that 237 packets out of the sample of 975 were under 125 g. The question was then posed: "Do these two samples suggest that the proportion of underweight packets is changing?"
Stating appropriate evidence, provide an answer to this question. [3]

$\hat{p}$ of new sample $= \dfrac{237}{975}$

$= 0.2431$

As the new $\hat{p}$ lies above the 97% C.I it appears the proportion of underweight packets is <u>increasing</u>, although this could be due to a difference in samples.